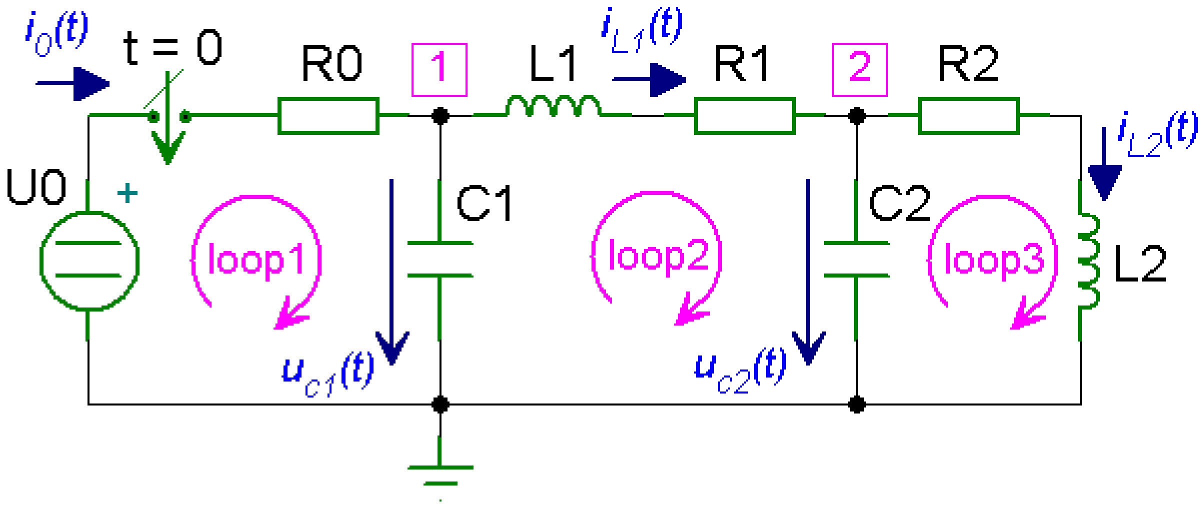
Transient Analysis

Example

For the circuit shown below find the currents $i_{L1}(t)$, $i_{L2}(t)$ and the voltages $u_{C1}(t)$, $u_{C2}(t)$. Use a numerical solution to the differential equations, when R_0 = 100 kΩ, R_1 = 300 Ω, R_2 = 10 kΩ, L_1 = 0.01 mH, L_2 = 10 mH, C_1 = 10 µF, C_2 = 10 nF, and U_0 = 10 kV. The switch has been opened for a long time.



Circuit

Solution

At $t < 0$ the voltages at the capacitors are

$$u_{C1}(0) = 0 \text{ V}, \qquad u_{C2}(0) = 0 \text{ V}$$

The currents flowing through the inductors are

$$i_{L1}(0) = 0 \text{ A}, \qquad i_{L2}(0) = 0 \text{ A}$$

At $t \geq 0$, the switch is closed. Using Kirchhoff's voltage law, for the loop 1 we have

$$R_0\, i_0(t) + u_{C1}(t) - U_0 = 0 \qquad => \qquad i_0(t) = \frac{U_0 - u_{C1}(t)}{R_0}$$

The other loop equations are:

Loop 2: $$L_1 \frac{di_{L1}(t)}{dt} + R_1\, i_{L1}(t) + u_{C2}(t) - u_{C1}(t) = 0$$

Loop 3: $$R_2\, i_{L2}(t) + L_2 \frac{di_{L2}(t)}{dt} - u_{C2}(t) = 0$$

Using Kirchhoff's current law and assuming that the currents leaving a node are positive, we have

For node 1:

$$-\frac{U_0 - u_{C1}(t)}{R_0} + C_1 \frac{du_{C1}(t)}{dt} + i_{L1}(t) = 0$$

At node 2:

$$-i_{L1}(t) + C_2 \frac{du_{C2}(t)}{dt} + i_{L2}(t) = 0$$

Simplifying, we get

$$\frac{di_{L1}(t)}{dt} = -\frac{R_1 i_{L1}(t)}{L_1} + \frac{u_{C1}(t)}{L_1} - \frac{u_{C2}(t)}{L_1}$$

$$\frac{di_{L2}(t)}{dt} = -\frac{R_2 i_{L2}(t)}{L_2} + \frac{u_{C2}(t)}{L_2}$$

$$\frac{du_{C1}(t)}{dt} = -\frac{i_{L1}(t)}{C_1} - \frac{u_{C1}(t)}{R_0 C_1} + \frac{U_0}{R_0 C_1}$$

$$\frac{du_{C2}(t)}{dt} = \frac{i_{L1}(t)}{C_2} - \frac{i_{L2}(t)}{C_2}$$

In matrix form, we have

$$\begin{bmatrix} \frac{di_{L1}(t)}{dt} \\ \frac{di_{L2}(t)}{dt} \\ \frac{du_{C1}(t)}{dt} \\ \frac{du_{C2}(t)}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R_1}{L_1} & 0 & \frac{1}{L_1} & -\frac{1}{L_1} \\ 0 & -\frac{R_2}{L_2} & 0 & \frac{1}{L_2} \\ -\frac{1}{C_1} & 0 & -\frac{1}{R_0 C_1} & 0 \\ \frac{1}{C_2} & -\frac{1}{C_2} & 0 & 0 \end{bmatrix} \begin{bmatrix} i_{L1}(t) \\ i_{L2}(t) \\ u_{C1}(t) \\ u_{C2}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{U_0}{R_0 C_1} \\ 0 \end{bmatrix}$$

The MATLAB program for solving the above differential equations in the interval $0 \le t \le 500\,\text{ms}$ is

MATLAB Script

```
function transient_analysis_03
% Transient analysis of RLC circuit using ode function
ts=0;     % start time
te=0.5;   % end time
x0=[0 0 0 0];  % initial conditions
[t,x]=ode23t(@diffeq,[ts te],x0);
% plot iL1(t)
subplot(4,1,1); plot(t,x(:,1));
xlabel('t [s]'); ylabel('iL1(t) [A]'); grid on;
% plot iL2(t)
subplot(4,1,2); plot(t,x(:,2));
xlabel('t [s]'); ylabel('iL2(t) [A]'); grid on;
% plot uC1(t)
subplot(4,1,3); plot(t,x(:,3));
xlabel('t [s]'); ylabel('uC1(t) [V]'); grid on;
% plot uC2(t)
subplot(4,1,4); plot(t,x(:,4));
xlabel('t [s]'); ylabel('uC2(t) [V]'); grid on;
% differential equations
function dxdt=diffeq(t,x)
R0=100e+3; R1=300; R2=10e+3;
L1=0.01e-3; L2=10e-3;
C1=10e-6; C2=10e-9;
U0=10e+3;
A=[ -R1/L1       0        1/L1          -1/L1;
       0      -R2/L2       0             1/L2;
     -1/C1       0       -1/(R0*C1)      0;
      1/C2     -1/C2       0             0 ];
F=[0; 0; U0/(R0*C1); 0];
dxdt=A*[x(1); x(2); x(3); x(4)]+F;
```

Notice:
If you used the function ode23, the computation of this task would be very time-consuming. Therefore the function ode23t has been used.

The plots obtained from MATLAB are