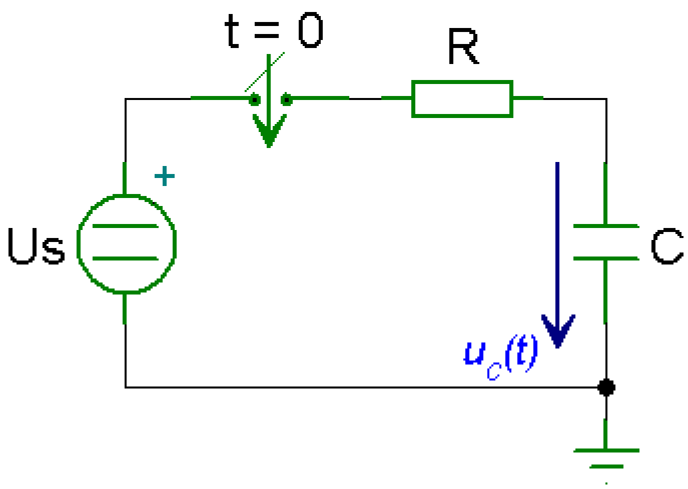
Transient Analysis

Example

For the circuit shown below find the voltage $u_C(t)$ between the interval 0 to 0.4 s, assuming that $u_C(0) = 0$. Use a numerical solution to the differential equations and analytical solution, when $R = 10\ \text{k}\Omega$, $C = 10\ \mu\text{F}$ and $U_S = 10$ V.



Circuit

Solution

At $t < 0$ the voltage at the capacitor is

$$u_C(0) = 0 \text{ V}$$

At $t \geq 0$, the switch is closed. Using Kirchhoff's voltage law, we have

$$R\,i(t) + u_C(t) - U_S = 0$$

$$RC\frac{du_C(t)}{dt} + u_C(t) = U_S$$

If the capacitor is initially uncharged, the analytical solution to the differential equation above is given as

$$u_C(t) = U_S\left(1 - e^{-\frac{t}{\tau}}\right)$$

where τ is the time constant, $\tau = R\,C$

From the differential equation, we get

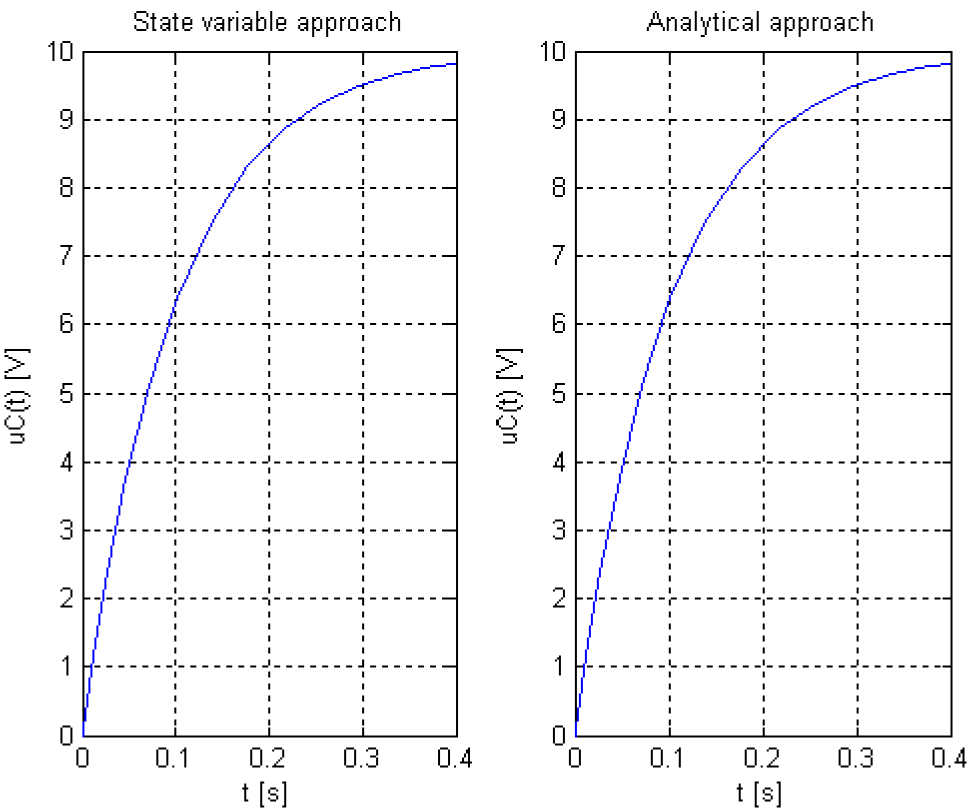
$$\frac{du_C(t)}{dt} = -\frac{u_C(t)}{RC} + \frac{U_S}{RC}$$

The MATLAB program for solving this task is

MATLAB Script

```
function transient_analysis_02
% Transient analysis of RC circuit
ts=0;   % start time
te=0.4; % end time
x0=0;   % initial condition
[t,x]=ode23(@diffeq,[ts te],x0);
% numerical solution
subplot(1,2,1); plot(t,x);
xlabel('t [s]'); ylabel('uC(t) [V]');
title('State variable approach'); grid on;
% analytical solution
R=10000; C=10e-6; Us=10;
uc=Us*(1-exp(-t/(R*C)));
subplot(1,2,2); plot(t,uc);
xlabel('t [s]'); ylabel('uC(t) [V]');
title('Analytical approach'); grid on;
% differential equation
function dxdt=diffeq(t,x)
R=10000; C=10e-6; Us=10;
A=-1/(R*C);
F=Us/(R*C);
dxdt=A*x+F;
```

The plots obtained from MATLAB are



From the two plots we can see that the two results are identical.