Transfer Functions

Example

Using MATLAB find the inverse Laplace transform of

$$G(s) = \frac{10s^2 + 20s + 40}{s^3 + 12s^2 + 47s + 60}$$

Solution

The MATLAB function *residue* can be used to perform a partial fraction expansion. Since $G(s)$ may represent an improper fraction, we may express $G(s)$ as a mixed fraction

$$G(s) = \frac{B(s)}{A(s)}$$

$$G(s) = \frac{N(s)}{D(s)} + \sum_{n=0}^{N} k_n s^n$$

where

$$\frac{N(s)}{D(s)} \text{ is a proper fraction}$$

We have

$$G(s) = \frac{r_1}{s - p_1} + \frac{r_2}{s - p_2} + \cdots + \frac{r_m}{s - p_m} + \sum_{n=0}^{N} k_n s^n$$

Given the coefficients of the numerator and denominator polynomials, the MATLAB *residue* function provides the values of r_1, r_2, … r_m and p_1, p_2, … p_m and k_1, k_2, … k_N. The general form of the *residue* function is

[**r**, **p**, **k**] = *residue*(**num**, **den**)

Vectors **num** and **den** specify the coefficients of the numerator and denominator polynomials in descending powers of s.

The residues are returned in the column vector **r**, the pole locations in the column vector **p**, and the direct terms in the row vector **k**.

For more information write the command *help residue* in the MATLAB command window.

The short MATLAB program for solving this task is

MATLAB Script

```
% The partial fraction expansion
numerator=[10 20 40];
denominator=[1 12 47 60];
[r,p,k]=residue(numerator,denominator)
```

The results obtained from MATLAB are

```
r =

   95.0000
 -120.0000
   35.0000


p =

   -5.0000
   -4.0000
   -3.0000


k =

     []
```

From the MATLAB results, we get

$$G(s) = \frac{95}{s+5} - \frac{120}{s+4} + \frac{35}{s+3}$$

and the inverse Laplace transform is

$$g(t) = 95\,e^{-5t} - 120\,e^{-4t} + 35\,e^{-3t}$$