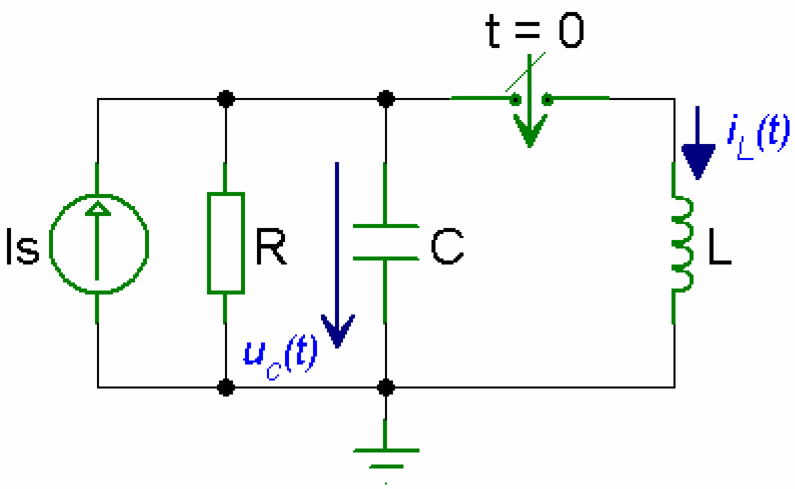
Transfer Functions

Example

For the circuit shown below find the response for the voltage $u_C(t)$, if $R = 10\ \Omega$, $L = 31.25$ mH, $C = 50\ \mu$F and $I_S = 2$ A. The switch has been opened for a long time.



Circuit

Solution

At $t < 0$ the voltage at the capacitor is

$$u_C(0) = R\, I_S = 20 \text{ V}$$

The current flowing through the inductor is

$$i_L(0) = 0 \text{ A}$$

At $t \geq 0$, the switch is closed. Using Kirchhoff's current law and assuming that the currents leaving the node are positive, for the node we get

$$-I_S + \frac{u_C(t)}{R} + C\frac{du_C(t)}{dt} + i_L(t) = 0$$

Thus

$$-I_S + \frac{u_C(t)}{R} + C\frac{du_C(t)}{dt} + \frac{1}{L}\int_0^t u_C(\tau)d\tau + i_L(0) = 0$$

Taking the Laplace transform of the above expression, we get

$$-\frac{I_S}{s}+\frac{U_C(s)}{R}+C[s\,U_C(s)-u_C(0)]+\frac{U_C(s)}{s\,L}+\frac{i_L(0)}{s}=0$$

Used Laplace transform pairs

K	$\frac{K}{s}$
$\frac{df(t)}{dt}$	$s\,F(s)-f(0)$
$\int_0^t f(\tau)d\tau$	$\frac{F(s)}{s}$

Thus

$$\frac{U_C(s)}{R}+C\,s\,U_C(s)+\frac{U_C(s)}{s\,L}=\frac{I_S}{s}+C\,u_C(0)-\frac{i_L(0)}{s}$$

$$L\,s\,U_C(s)+R\,L\,C\,s^2\,U_C(s)+R\,U_C(s)=R\,L\,I_S+R\,L\,C\,s\,u_C(0)-R\,L\,i_L(0)$$

$$U_C(s)=\frac{R\,L\,I_S+R\,L\,C\,s\,u_C(0)-R\,L\,i_L(0)}{L\,s+R\,L\,C\,s^2+R}$$

Applying the initial conditions, we get

$$U_C(s)=\frac{20R\,L\,C\,s+R\,L\,I_S}{R\,L\,C\,s^2+L\,s+R}$$

The short MATLAB program for solving this task is

MATLAB Script

```
clear; clc;
R=10; L=31.25e-3; C=50e-6; Is=2;
ts=0;      % start time
te=30e-3;  % end time
numerator=[20*R*L*C  R*L*Is];
denominator=[R*L*C  L  R];
[r,p,k]=residue(numerator,denominator)
t=ts:te/100:te;
u=r.'*exp(p*t);  % r.' is the non-conjugate transpose
plot(t,u);
xlabel('t [s]'); ylabel('uC(t) [V]'); grid on;
```

The results obtained from MATLAB are

```
r =
   -6.6667
   26.6667

p =
   -1600
   -400

k =
      []
```

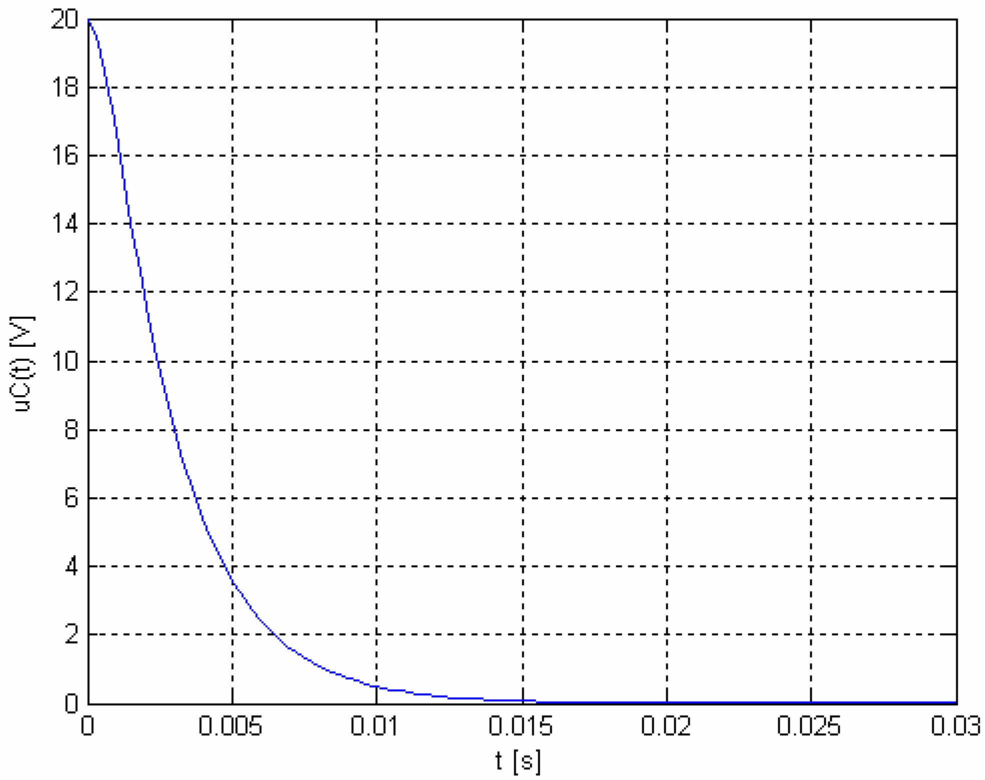
From the MATLAB results, we get

$$U_C(s) = -\frac{6.67}{s+1600} + \frac{26.67}{s+400}$$

and the inverse Laplace transform is

$$u_C(t) = -\,6.67\, e^{-1600\,t} + 26.67\, e^{-400\,t} \quad [\mathrm{V}]$$

The plot obtained from MATLAB is



<u>Notice</u>: This plot and the plot obtained from the example with the same circuit solved by a numerical method are identical. You can compare these plots.