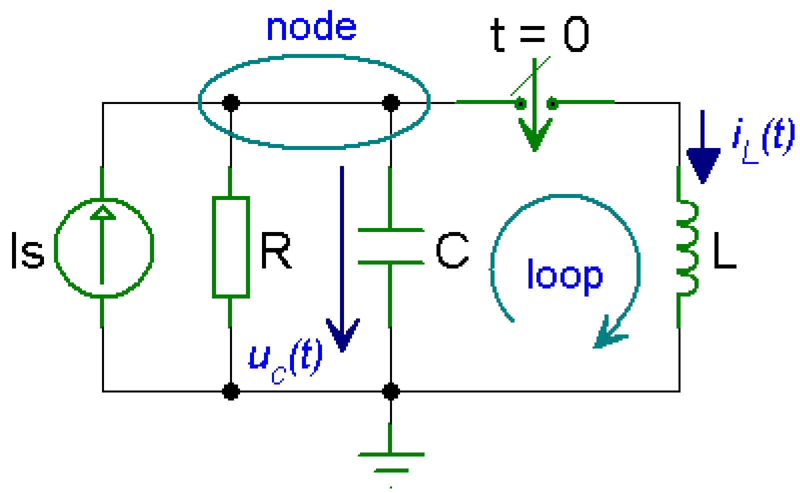
Transient Analysis

Example

For the circuit shown below find the current $i_L(t)$ and the voltage $u_C(t)$. Use a numerical solution to the differential equations, when $R = 10\ \Omega$, $L = 31.25$ mH, $C = 50\ \mu$F and $I_S = 2$ A. The switch has been opened for a long time.



Circuit

Solution

At $t < 0$ the voltage at the capacitor is

$$u_C(0) = R\, I_S = 20 \text{ V}$$

The current flowing through the inductor is

$$i_L(0) = 0 \text{ A}$$

At $t \geq 0$, the switch is closed and all the four elements of the circuit remain parallel. Using Kirchhoff's voltage law, for the loop we have

$$L\frac{di_L(t)}{dt} - u_C(t) = 0$$

Using Kirchhoff's current law and assuming that the currents leaving the node are positive, for the node we get

$$-I_S + \frac{u_C(t)}{R} + C\frac{du_C(t)}{dt} + i_L(t) = 0$$

Simplifying, we get

$$\frac{di_{\mathrm{L}}(t)}{dt} = \frac{u_{\mathrm{C}}(t)}{L}$$

$$\frac{du_{\mathrm{C}}(t)}{dt} = -\frac{i_{\mathrm{L}}(t)}{C} - \frac{u_{\mathrm{C}}(t)}{RC} + \frac{I_{\mathrm{S}}}{C}$$

In matrix form, we have

$$\begin{bmatrix} \dfrac{di_{\mathrm{L}}(t)}{dt} \\ \dfrac{du_{\mathrm{C}}(t)}{dt} \end{bmatrix} = \begin{bmatrix} 0 & \dfrac{1}{L} \\ -\dfrac{1}{C} & -\dfrac{1}{RC} \end{bmatrix} \begin{bmatrix} i_{\mathrm{L}}(t) \\ u_{\mathrm{C}}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{I_{\mathrm{S}}}{C} \end{bmatrix}$$

The MATLAB program for solving the above differential equations in the interval $0 \le t \le 30\,\mathrm{ms}$ is

MATLAB Script

```
function transient_analysis_01
% Transient analysis of RLC circuit using ode function
ts=0;      % start time
te=30e-3;  % end time
x0=[0 20];  % initial conditions
[t,x]=ode23(@diffeq,[ts te],x0);
% plot iL(t)
subplot(2,1,1); plot(t,x(:,1));
xlabel('t [s]'); ylabel('iL(t) [A]'); grid on;
% plot uC(t)
subplot(2,1,2); plot(t,x(:,2));
xlabel('t [s]'); ylabel('uC(t) [V]'); grid on;
% differential equations
function dxdt=diffeq(t,x)
R=10; L=31.25e-3; C=50e-6; Is=2;
A=[0        1/L;
   -1/C    -1/(R*C)];
F=[0; Is/C];
dxdt=A*[x(1); x(2)]+F;
```

The plots obtained from MATLAB are