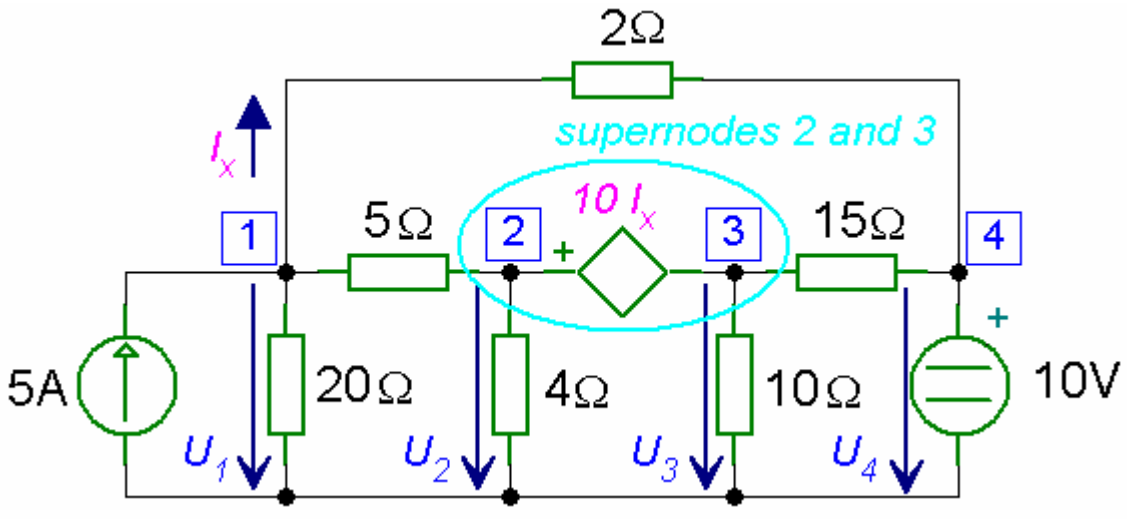
Nodal Analysis (Dependent Source)

Example

For the circuit shown below find the nodal voltages U_1, U_2, U_3 and U_4.



Circuit with Dependent and Independent Sources

Solution

Using Kirchhoff's current law and the convention that the currents leaving a node are positive, we have

For node 1:

$$\frac{U_1}{20} + \frac{U_1 - U_2}{5} + \frac{U_1 - U_4}{2} - 5 = 0$$

$$0.05\,U_1 + 0.2\,U_1 - 0.2\,U_2 + 0.5\,U_1 - 0.5\,U_4 - 5 = 0$$

$$0.75\,U_1 - 0.2\,U_2 - 0.5\,U_4 = 5$$

From supernodes 2 and 3, we have:

$$-\frac{U_1 - U_2}{5} + \frac{U_2}{4} + \frac{U_3}{10} + \frac{U_3 - U_4}{15} = 0$$

$$-0.2\,U_1 + 0.2\,U_2 + 0.25\,U_2 + 0.1\,U_3 + 0.06667\,U_3 - 0.06667\,U_4 = 0$$

$$-0.2\,U_1 + 0.45\,U_2 + 0.16667\,U_3 - 0.06667\,U_4 = 0$$

At node 4:

$$U_4 = 10$$

For the dependent source:

$$U_2 - U_3 = 10\, I_x\,, \quad \text{but} \quad I_x = \frac{U_1 - U_4}{2}$$

Thus

$$U_2 - U_3 = \frac{10(U_1 - U_4)}{2}$$

Simplifying, we get

$$-5\, U_1 + U_2 - U_3 + 5\, U_4 = 0$$

In matrix form, we have

$$\begin{bmatrix} 0.75 & -0.2 & 0 & -0.5 \\ -0.2 & 0.45 & 0.16667 & -0.06667 \\ 0 & 0 & 0 & 1 \\ -5 & 1 & -1 & 5 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 10 \\ 0 \end{bmatrix}$$

The MATLAB program for solving the nodal voltages is

MATLAB Script

```
% this program computes the nodal voltages
% given the admittance matrix Y and current vector I
% Y is the admittance matrix
% I is the current vector
% U is the nodal voltage vector
% initialize matrix Y and vector I using YU=I form
Y = [0.75   -0.2     0   -0.5;
     -0.2   0.45  0.166666667  -0.0666666667;
        0      0     0     1;
       -5      1    -1     5];
I = [5; 0; 10; 0];
% solution for the voltages
disp('The nodal voltages U1, U2, U3 and U4 are')
U = inv(Y)*I
```

The results obtained from MATLAB are

```
The nodal voltages U1, U2, U3 and U4 are
U =
   18.1107
   17.9153
  -22.6384
   10.0000
```